博士（理学）　王　　海　兵

学 位 論 文 題 名

Studies on Stationary and Dynamical Inverse Problems

（定常及び非定常逆問題の研究）

学位論文内容の要旨

In this dissertation, we study some stationary and dynamical inverse problems. More precisely, in the first part, we mainly focus on the inverse scattering of obliquely incident time-harmonic electromagnetic waves by an impedance cylinder. For this purpose, we shall start with the corresponding forward scattering problem. Then, in the second part, we are concerned with the so-called thermography which is modeled by an inverse boundary value problem for the heat equation. The outline of dissertation is as follows:

In Chapter 1, we give an introduction on (forward and inverse) scattering problem and thermography with detailed mathematical descriptions. We will also review some well-known results related to our works presented in the following chapters. Then we propose our studied problems and state the results.

In Chapter 2, we consider the scattering of obliquely incident time-harmonic electromagnetic waves by an impedance cylinder which is embedded in an inhomogeneous medium. We assume that the cylinder and the medium are uniform along the x_3 axis of cylinder. Also, the inhomogeneity of the medium with respect to (x_1, x_2) is confined to a large enough bounded set in the (x_1, x_2)-space. Since the x_1 components and x_2 components of electric field and magnetic field can be expressed in terms of their x_3 components, we can derive from Maxwell's equations and Leontovich impedance boundary condition that our scattering problem is modeled as a boundary value problem for a second-order elliptic system with oblique boundary condition. Using Rellich's lemma, the uniqueness of solutions to the boundary value problem is justified. To show the existence of its solution, the Lax-Phillips method is used. The key point for that is to prove the solvability of the associated oblique derivative problem in a bounded domain consisting of two boundaries which are the boundary of the cross-section of the cylinder with coupled oblique boundary condition and that of a domain containing this cross-section with Dirichlet boundary condition.

In Chapter 3, we study a special case of the above scattering problem, that is, the background medium exterior to the cylinder is homogeneous. In this setting, the scattering problem is governed by a system of two Helmholtz equations with coupled oblique boundary conditions. Using the boundary integral equation method, we show that the scattering problem is uniquely solvable. Indeed, by expressing the scattered fields in the form of single-layer potentials, our oblique scattering problem is reformulated as a system of two integral equations. But it is not a usual Fredholm system of the second kind as that in the case of normal incidence, since the system involves the tangential derivatives of the single-layer potential. By relating it to the Cauchy integral operator, we show that this system of integral operators is of Fredholm type with index 0. Therefore, the solvability of the integral system follows from the uniqueness of its solutions due to the Fredholm theory. A numerical scheme for solving the integral equations is also presented with some numerics. The numerical results illustrate the validity and efficiency of the proposed method.

In Chapter 4, we investigate an inverse scattering problem for obliquely incident electromagnetic waves, where we assume that the host medium is homogeneous. More precisely, we are concerned with the inverse problem of uniquely identifying the cross-section of an unknown cylinder and the impedance function from the far-field patterns at fixed frequency and a range of incident angles. A uniqueness result for such an inverse scattering problem is established. Our method is based on the analyticity of solution to the direct scattering problem, which is justified by using the Lax-Phillips method together with the perturbation theory of Fredholm operators.

In Chapter 5, we are concerned with an inverse problem of identifying the unknown cylinder from the far-field measurements of polarized electromagnetic plane waves. By using the generalized mixed reciprocity principle combined with the singularity analysis of the so-called reflected solutions for point sources, we show that the cross-section of the cylinder can be uniquely determined from the far-field patterns of only the electric or magnetic field, through the use of special polarized incident plane waves. We emphasize that our result would be helpful in practical situations where only one type of measurements is available. Moreover, our argument is reconstructive and it can be directly used to establish a reconstruction scheme. In fact, the singular behavior of the reflected solutions for point sources enables us to reconstruct the shape of the cross-section numerically. To recover the impedance, we need to use a more singular source.

In Chapter 6, we consider an inverse problem of identifying the unknown cavities in a heat conductor, which typically models the thermography. Taking the Neumann-to-Dirichlet map as measured data, we develop a linear sampling type method for the heat equation. A new feature is that there is a freedom to choose the time variable, which suggests that we have more data than the linear sampling methods for the inverse boundary value problem associated with electrical impedance tomography and inverse scattering problem with near-field data.

学位論文審査の要旨

主　査　　教　授　　中　村　　　玄
副　査　　教　授　　神　保　秀　一
副　査　　教　授　　利根川　吉　廣

学　位　論　文　題　名

Studies on Stationary and Dynamical Inverse Problems

（定常及び非定常逆問題の研究）

博士学位論文審査等の結果について（報告）

本論文は、(i)z 軸方向に一様な媒質中におかれたインピーダンスを持った柱状体による時間周期的な電磁波の散乱とその逆問題及び(ii)伝熱体中の未知空洞同定境界値逆問題に対する linear sampling 法に関する研究である。何れも応用上重要な非破壊検査法の数理に対する研究であるが、本論文はこれらの研究に対して確固とした新しい研究領域を開拓したと言える。各研究テーマの概要は、次の通りである。(i)は所謂偏極されたTE 或いはTM 電磁波平面波が柱状体の中心軸に対して必ずしも垂直でない方向から入射した場合の電磁波の散乱問題の適切性の問題とこれらの入射波に対応する散乱波の遠方場を計測して柱状体の断面形状とインピーダンスを求める逆問題に関するものである。

(i)、(ii)の研究については、応用上非常に重要であるにもかかわらずその難しさの為に長い間数学研究がなされて来なかった。本論文は、まず適切性の問題に完全な解答を与え、それをもとに次の二種類の逆問題を考察している。まず、入射波の入射方向と z 軸のなす角 θ を 90° を含む様な任意の開区間で変動して得られる TE, TM 入射電磁平面波に対する両方の散乱波の遠方場から柱状体の断面形状を同定する新しい逆問題を提出し、その同定が一意的であることを示している。次に TE 或いはTM 電磁平面波の何れか一方を角 θ は変えずに入射方向と x 軸、y 軸とのなす方向が全ての方向になる様に入射させたとき、それに対応する TE, TM 散乱波の遠方場だけを計測して柱状体の断面形状とインピーダンスの再構成を、singular sources 法を用いて示している。(ii)については散乱の逆問題の解法として最も有名な linear sampling 法を、伝熱体中の未知空洞同定逆問題に対して確立した。これはサーモグラフィーに対する極めて簡便で新しい逆解析手法を与えるものであり、今後この研究が様々な形で発展する出発点を与えた重要な研究である。

(i)と(ii)の何れの研究も既に定評のある国際誌に掲載あるいは受理されている。従って著者は、北海道大学博士（理学）の学位を授与される資格あるものと認める。